

Parity simulation of thermoacoustic effect in regenerator of Stirling cryocooler

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The thermally induced acoustic effect occurring in a regenerator with a temperature gradient has been observed on a particularly built regenerator experimental set-up attached to a Stirling cryocooler system. To describe this thermoacoustic phenomenon, a parity simulation network model has been established in the form of an oscillation circuit with positive feedback amplification. The model vividly shows the influence on the whole system of the change in flow impedance in the regenerator caused by the change in temperature gradient.

Keywords: thermoacoustics; parity simulation model; regenerator; positive feedback amplifier

Nomenclature

A	Cross-sectional area (m^2)
C	Volume capacitance of fluid ($\text{m}^5 \text{N}^{-1}$)
C_d	Motion damping factor of displacer (N s m^{-1})
F	Amplification ratio of feedback circuit
G	Voltage ratio of secondary in transformer
I	Equivalent current in network
K	Rigidity of spring of displacer (N m^{-1})
L	Equivalent inductance of fluid ($\text{N s}^2 \text{m}^{-5}$)
M	Mass of displacer (kg)
P	Pressure wave (N m^{-2})
Q	Flow flux ($\text{m}^3 \text{s}^{-1}$)
R	Equivalent resistance of fluid (N s m^{-5})
R_g	Gas constant ($\text{m}^2 \text{s}^{-2} \text{K}^{-1}$)
s	Complex frequency for Laplace transformation
T	Temperature (K)
U	Equivalent voltage in network
V	Volume (m^3)
X	Displacement of displacer (m)
Z	Equivalent impedance of regenerator

Superscripts

,	Composite
.	Derivative of first order
..	Derivative of second order
→	Phasor

Subscripts

ab, bc	Two parts of secondary in voltage transformer
e, w, d	Cold space, warm space and displacer of cryocooler
f	Feedback
g	Pneumatic space in cryocooler
i, o	Inlet and outlet
m	Matrix of regenerator
r	Regenerator
rod	Driving rod of displacer

Greek letters

β	Amplification gain
ΔT_r	Longitudinal temperature difference in regenerator (K)
ϵ	Filling factor of matrix
ω	Angular frequency

From a thermoacoustic point of view, it is now being recognized more and more commonly that in a regenerative heat engine the conversion between heat and work is essentially carried out by the coupling of a longitudinal pressure wave propagating in the working fluid with a longitudinal temperature gradient of the matrix via transverse heat transfer in the regenerator, i.e. thermoacoustic conversion by the regenerator. This causes, in the case of a positive temperature gradient along the line of pressure propagation, the amplification of acoustic energy, while it generates heat pumping in the case of a negative temperature gradient¹⁻³. In more detail, the thermoacoustic effect in a regenerator is described as the increment of acoustic energy in the direction of the temperature gradient due to heat addition by a transverse temperature wave, modulated by gas parcel displacement along with the propagation of a pressure wave⁴. After successful modelling of pressure wave propagation in an isothermal regenerator by a passive network of fluid flow reactance (see *Figure 1a*)⁵, recently the thermoacoustic conversion in a regenerator has been characterized by an active network made up of acoustic impedance and a source element reflecting the contribution of gas parcel velocity variation during its reciprocating displacement along the longitudinal temperature gradient (see *Figure 1b*)⁶.

It has also been found experimentally that, in a split cycle Stirling cryocooler, the cold space temperature significantly influences the working process of the cryocooler system^{7,8}. From the thermoacoustic point of view, this effect reflects the interaction between the longitudinal temperature gradient of the matrix and wave propagation in the working gas. A detailed investigation would reveal more about the mechanism of conversion between heat and work in the regenerator. For this purpose, a series of experiments have been carried out on a specially designed split cycle Stirling cryocooler to investigate the mechanism of the conversion. Based on network modelling of the regenerator, the idea of parity simulation proposed first at MIT⁹ has been developed to apply to a parity simulation circuit for a regenerator. The endeavour to combine the thermoacoustic viewpoint with the fluid network modelling theory¹⁰ to provide a more con-

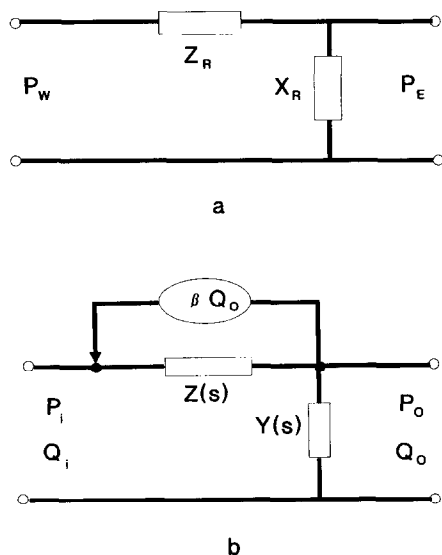


Figure 1 Equivalent network of wave propagation for regenerator. (a) Passive form for isothermal system; (b) active form for regenerator with temperature gradient

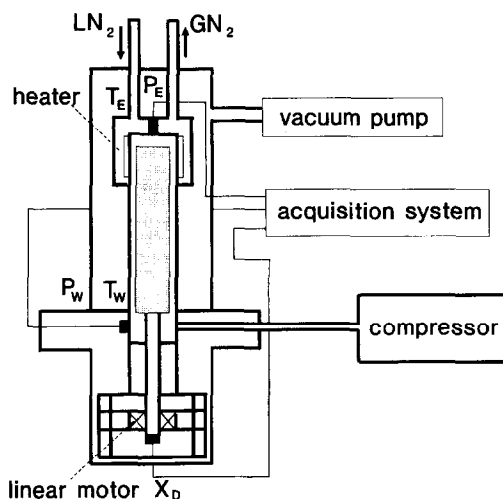


Figure 2 Schematic diagram of experimental set-up

venient engineering tool is also one of the objectives of this paper.

Experimental set-up and main results

An experimental split Stirling cryocooler (shown in *Figure 2*) with a controllable displacer (driven by a linear motor) and an adjustable longitudinal temperature difference ΔT_r exerted between the ends of the regenerator, has been built specially for this investigation. The temperature in cold space T_c can be determined by a liquid nitrogen source and an electric heater both pressed tightly against the head cup of the cold space. The linear motor attached to the displacer is fed with square wave signals by a frequency regulator. During the experiments, by controlling the temperature T_c and driving the displacer to move in a sinusoidal pattern, the pressure waves in the cold and warm spaces P_c , P_w and the displacement of the displacer X_d , have been measured accurately under various temperature differences ΔT_r . All these steps have been carried out under the condition that the compressor is out of operation. Two examples of the measured pressure and displacement waves with a selected temperature difference ΔT_r , after the noise being filtered, are shown in *Figures 3* and *4*, while all the results are summarized in *Figures 5* and *6*. The experimental results reveal the following main facts.

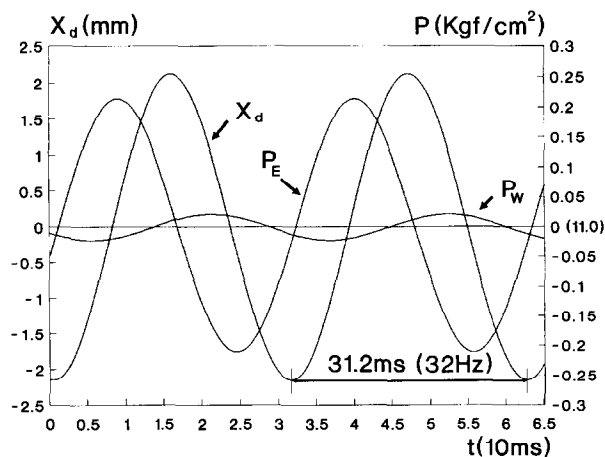


Figure 3 Results of X_d , P_e and P_w fluctuation (after filtration) measured at $\Delta T_r = 0$ K ($T_e = T_w = 300$ K)

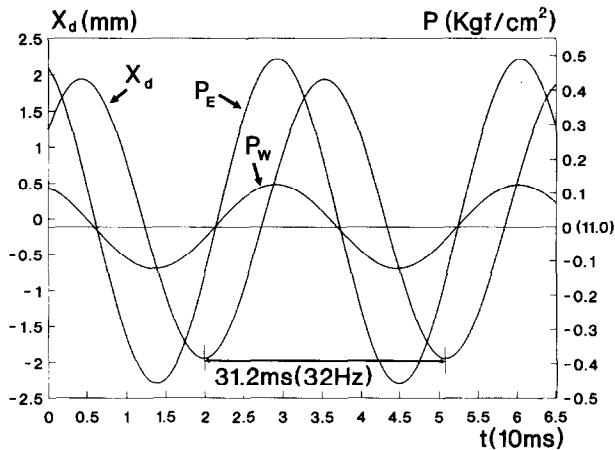


Figure 4 Results of X_d , P_E and P_W fluctuation (after filtration) measured at $\Delta T_R = 163$ K ($T_E = 122$ K, $T_W = 285$ K)

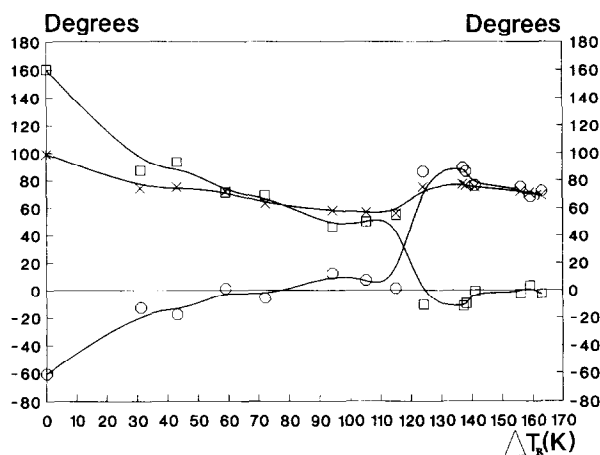


Figure 5 Phase shift between X_d , P_E and P_W in terms of ΔT_R . $\square$, $P_E \angle P_W$; $\circ$, $P_W \angle X_d$; $\times$, $P_E \angle X_d$

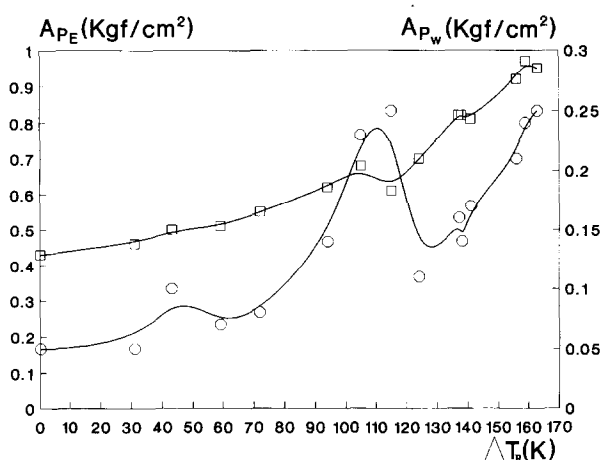


Figure 6 Amplitude variation of P_E and P_W with ΔT_R . $\square$, A_{P_E} ; $\circ$, A_{P_W} (charging pressure = 11.0 kgf cm^{-2})

The longitudinal temperature difference ΔT_r imposes a significant influence on the pressure wave propagation induced by the motion of the displacer in the regenerator, which can be characterized not only by phase shifts between P_e , P_w and X_d , but also by the increment of the amplitudes of P_e and P_w . Specifically, at zero ΔT_r , P_e has a phase lead over P_w of $\approx 180^\circ$ (shown in Figure 3), and the amplitude of P_w is very small compared to P_e because

the warm space is connected with the compression space by a linking pipe and appears to be almost the full portion of the dead volume of the cryocooler system, while the volume of the cold space occupies only a very small portion of it. With the appearance of ΔT_r , however, the phase lead of P_e over P_w drops rapidly, and P_e goes ahead of P_w by $\approx 90^\circ$ even if the ΔT_r is small (shown in Figure 5). During the process of ΔT_r changing from 0 to 110 K, the phase difference between P_e and P_w reduces slowly from $\approx 90^\circ$ to 60° , whereas the amplitudes of P_w and P_e (especially P_w) obviously increase (shown in Figures 5 and 6). When the temperature difference ΔT_r attains the level of 120 K, the phase of P_w shifts abruptly and comes into phase with P_e . To compensate the violent phase shifting of P_w , its amplitude reduces.

As the temperature difference ΔT_r continues to increase, the in-phase relationship between P_e and P_w does not obviously change and the amplitudes of these two pressure waves rise together. Thus there are two points where abrupt phase shifting between P_e , P_w and X_d (shown in Figure 5) occurs with a corresponding variation in the amplitudes of P_e and P_w (shown in Figure 6) in the experimental temperature difference ΔT_r range; and here the second point is more noteworthy. In contrast to the changing phase relation between P_e and P_w , the phase relation between P_e and X_d shows no obvious variation and P_e continues to have a phase lead over X_d of $\approx 90^\circ$ in the whole experimental range.

Parity simulation based on network modelling of regenerator

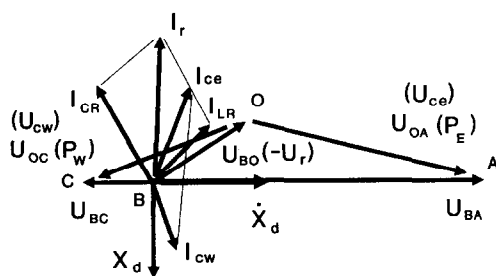
To simulate the wave process caused by the motion of the regenerator, the displacer's cylinder and the movable regenerator inside it can be described as a cylinder with a reciprocating piston in it, which divides the cylinder into two unequal separated spaces, V_w and V_e ($V_w \gg V_e$), both of them being connected by a bypass tube with a flow impedance equivalent to that of the regenerator. A closed-core transformer with a specific point tapped secondary can be used to model the pressure wave effect induced by the motion of the piston, and the displacement rate of the regenerator $\dot{X}_d$ is modelled as the current of the transformer's primary I_{xd} which provides two voltages U_{ab} and U_{cb} in the secondary, each with the opposite phase to the other to simulate the pressures P_w and P_e in the case of zero ΔT_r . According to the already known network expression for wave propagation in a stationary regenerator⁶, a parity circuit of the pressure wave effect produced by the motion of the regenerator can be illustrated as in Figure 7a, in which the tap point of the secondary of the transformer shifts to the end near C_w to give an amplitude of U_{ab} much larger than that of U_{cb} . Here, the capacitors C_e , C_r and C_w represent the mass accumulation capacity of the cold space, regenerator and warm space, respectively, and the inductor L_r and resistor R_r correspond to the effect of inertia and viscosity of the working fluid in the regenerator. The circuit gives the following relations

$$U_{ab} = U_{ce} + U_r \quad (1)$$

$$U_{cb} = U_{cw} + U_r \quad (2)$$

$$I_{ce} = I_r + I_{cw} \quad (3)$$

which agree with the conservation principle in a regenerator

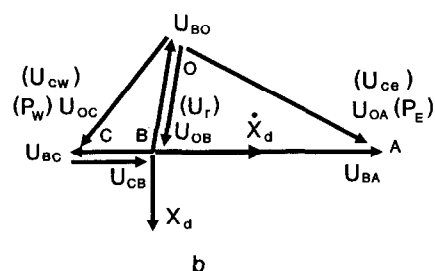


and two adjacent spaces. In the above equations, voltage U_r represents the pressure drop across the regenerator, and the currents I_{cc} , I_r and I_{cw} stand for the mass flow rate of gas in the three elements. Since U_{ab} is in phase with I_{xd} , in contrast U_{cb} , U_{cc} and U_{cw} are nearly out of phase depending on U_r in *Figure 7b*. Quantitatively, by applying the principle of equivalent transformation of a two-port active network to the calculation, the transfer function between U_{cc} and U_{cw} can be obtained as

here Z_r is the total impedance of the regenerator

and G is determined as

from which it can be seen that the phase relation between P_e and P_w is significantly determined by the impedance Z_r of the regenerator. In accordance with the fact that under $\Delta T_r = 0$ K the flow impedance of the regenerator is dominated by the fluid capacitance C_r (reference 5), so U_{cr} simulating the pressure drop component contributed by this capacitance should be larger than U_{Lr} caused by fluid inertia. Phasor analysis of the phase relation between U_{ce} , U_{cw} and I_{xd} in *Figure 7b* shows good agreement with the experimental situation where $\Delta T_r = 0$ K (seen in *Figure 3*).



flux is modulated by the pressure wave and this contributes to sound generation, i.e. pressure amplification under the condition of a positive temperature gradient. This coincides with the situation in the experiments, so it is clearly shown that, when the temperature gradient is raised, the amplitudes of P_e and P_w increase separately in varying degrees, while the disturbance X_d almost does not change as shown in *Figures 3* and *4*. This phenomenon is analogous to the situation in a oscillation circuit with a positive feedback amplifier, which modulates a d.c. voltage to an a.c. one, with its input being amplified at the same time. To describe this effect, the source in the network shown in *Figure 1b* is simulated by the positive feedback amplifier circuit shown in *Figure 8a*, in which the d.c. power source V_s corresponds to the heat input and its power level increases along with augmentation of the temperature gradient. The voltage drop U_{ce} on C_e is amplified by a two-stage amplifier whose output is fed to the parallel circuits B–O and C–O. A couple of transistors are needed to satisfy the phase relation for the positive feedback amplifier circuit because the output of a single transistor is in opposite phase to its input. The positive feedback loop is composed of the parallel circuit and part of the secondary of transformer A–B (shown in *Figure 9*), so that the output of the amplifier circuit is fed back to the input. From analysis of the positive feedback loop

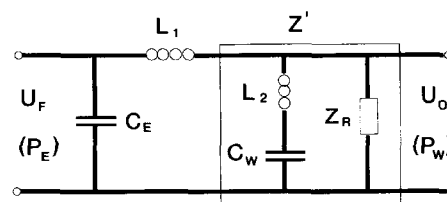


Figure 9 Positive feedback loop of circuit for regenerator with oscillation

$$\vec{F} = \frac{\vec{U}_r}{\vec{U}_o} = 1 + \frac{L_1}{Z'} \times s \quad (7)$$

can be obtained, in which impedance Z' includes the combination of Z_r , C_w and L_2 , while L_1 and L_2 are the inductance of the two parts of the secondary A-B, B-C. In an ideal case, $L_1 = L_2 = 0$, so $F = 1$. If the temperature difference across the regenerator is initially small, the gain of the amplifier circuit is strong enough to contend with the effect of the secondary of the transformer, and so the voltage relations (1) and (2) must be changed to (8) and (9)

$$U_{ce} = U_{ab} + U_r \quad (8)$$

$$U_r = U_{bc} + U_{cw} \quad (9)$$

which means that the amplifier circuit becomes a determining component in the power supply of the system. The phasor analysis shown in Figure 8b gives the phase relation between U_{ce} , U_{cw} and I_{xd} , which is in good agreement with the case of $0 < \Delta T_r < 120$ K. In the process where ΔT_r increases up to about 120 K, U_{ce} continues to be amplified to keep U_{cw} amplified, while the phase shift between U_{ce} and U_{cw} is slowly decreased. The reason behind this change will be given in a later analysis. Evidently, the increase of ΔT_r corresponds to the increment of heat input, i.e. augmentation of the mains voltage V_s in the feedback circuit.

Next, we go further into the situation where ΔT_r becomes larger than 120 K. In the network in Figure 8a, the flow impedance of the regenerator Z_r would change because the flow resistance of the matrix decreases continuously along with the cold space temperature reduction, while the flow inductance is steadily augmented and the fluid capacitance remains almost unchanged ($R_r \propto \mu$, $L_r \propto \rho$, $C_r \propto 1/(R_g \times P) \equiv \text{constant}$). When ΔT_r reaches ≈ 120 K, the flow inductance of the regenerator increases so as to be large enough to compensate thoroughly the effect of fluid capacitance; a kind of parallel resonance occurs in the regenerator which shows a relatively pure resistance characteristic. Thus the phase relation between U_{ce} and U_{cw} undergoes a sudden change, so that U_{cw} is in phase with U_{ce} according to the phasor analysis shown in Figure 10. So it can be concluded that, along with the establishment of a temperature gradient, the flow impedance of the regenerator varies gradually from the capacitance-dominated state to the resistance-dominated one. In the process where ΔT_r continues to increase, the resonance condition remains and the amplification effect steadily grows while the heat input increases. It is regrettable that the authors failed to complete the experiment reported above to attain a cold space temperature of 77 K. But from the results in another

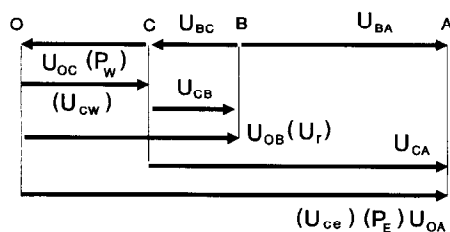


Figure 10 Vector analysis graph for regenerator network at its resonant state

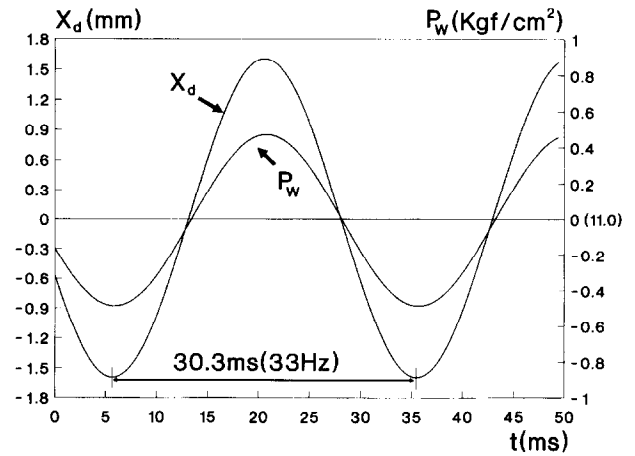


Figure 11 Results of P_w and X_d fluctuation measured at $T_E = 94$ K

similar experiment⁸ shown in Figure 11, it may be deduced that the departure from the resonance state would occur in the case of $T_e = 94$ K, corresponding to $\Delta T_r = 200$ K, by comparing previous values with the results in the range $0 < \Delta T_r < 120$ K.

Thus, a parity simulation circuit based on the thermoacoustic network modelling of a Stirling cryocooler regenerator has been established and shown to be in good agreement with the experimentally revealed effect of temperature gradient on pressure wave propagation.

Combining the analysis above with the results of another experiment reported in reference 8, in which the variation of the damping factor C_d of the displacer with respect to refrigeration temperature was determined by an on-line method⁷, i.e. measurement in a working Stirling cryocooler system, it can be seen that the dependence of the flow impedance of a regenerator on the cold space temperature is significantly relevant to the working performance of the cryocooler. One especially important result of the experiment in reference 8, the change in shape of the damping factor C_d , expressed in the motion equation (10), for the displacer as the refrigeration temperature was dropping

$$M \times \ddot{X}_d'' + C_d \times \dot{X}_d + K \times X_d = (P_g - P_w) \times A_{rod} + (P_w - P_c) \times (A_m \times \epsilon + A_d - A_m) \quad (10)$$

is shown in Figure 12. Obviously, the change in motion damping with cooling down has strong non-linear behavior.

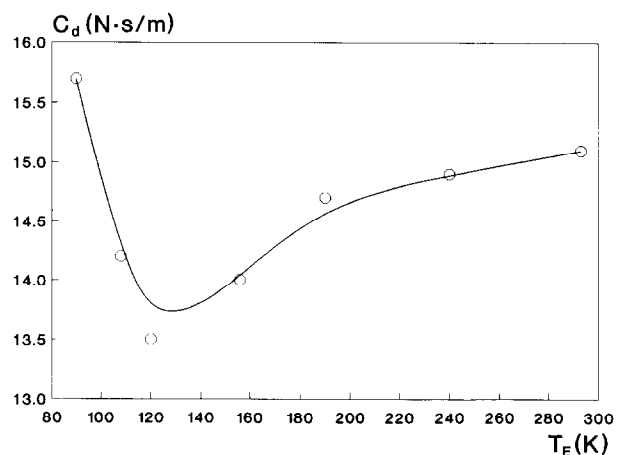


Figure 12 Changing pattern of C_d with T_E

ior, shown as the stepping down of damping when the refrigeration temperature is higher than 190 K, then the progressively accelerating decline until the minimum is reached at ≈ 130 K and, finally, the sudden step rise when the refrigeration temperature is below 120 K.

Based on the experimental facts presented in this paper and the above discussion, it is easy to understand the reason behind the singular change of damping. In fact, the motion damping changes along with the dropping refrigeration temperature in two opposite ways: one trend is to decrease, due to the monotonic decrease of gas viscosity, being represented as the stepwise reduction of flow resistance of the matrix in the regenerator; while another trend is to increase on the appearance of the activeness of the regenerator, especially when the resonance state of the regenerator occurs. Evidently, at a refrigeration temperature lower than 160 K ($\Delta T_r > 120$ K), the motion damping of the displacer is significantly determined by the state of flow impedance of the regenerator. From that, it can be seen that the activeness induced by the thermoacoustic effect in a regenerator is a key link between the so-called negative feedback relationship of the cold space temperature and the performance of the cryocooler, proposed first in reference 4.

Conclusions

From the experimental results and the above analysis, a thermally induced acoustic effect in a regenerator with a temperature gradient, namely the activeness of the regenerator, has been found and successfully simulated by a parity simulation circuit with positive feedback amplification. The activeness of the regenerator can be generated not only by the modulation of pressure wave propagation, e.g. in a stationary regenerator, but also by the motion of the regenerator, e.g. the regenerator in a Stirling cryocooler. Moreover, it would change along with the temperature gradient

in the form of variation of flow impedance of the regenerator, as observed in our experimental range where the flow impedance of the regenerator changes from the flow capacitance-dominated state to an important flow resistance-dominated resonance state as the temperature gradient across the regenerator steadily increases. As a result, the performance of the cryocooler has been greatly affected by this activeness.

Acknowledgements

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